Exercise 4

(a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

$$\frac{2}{x} - \frac{1}{y} = 4$$

Solution

Part (a)

Differentiate both sides with respect to x.

$$\frac{d}{dx}\left(\frac{2}{x} - \frac{1}{y}\right) = \frac{d}{dx}(4)$$

$$2\frac{d}{dx}(x^{-1}) - \frac{d}{dx}(y^{-1}) = 0$$

$$2(-x^{-2}) - \left[-y^{-2} \cdot \frac{d}{dx}(y)\right] = 0$$

$$-\frac{2}{x^2} + \frac{y'}{y^2} = 0$$

Solve for y'.

$$y' = \frac{2y^2}{x^2}$$

Part (b)

Solve for y first.

$$\frac{1}{y} = \frac{2}{x} - 4 = \frac{2-4x}{x}$$

$$y = \frac{x}{2-4x}$$

Then take the derivative.

$$y' = \frac{d}{dx}\left(\frac{x}{2-4x}\right) = \frac{\left[\frac{d}{dx}(x)\right](2-4x) - x\left[\frac{d}{dx}(2-4x)\right]}{(2-4x)^2} = \frac{(1)(2-4x) - x(-4)}{(2-4x)^2} = \frac{2}{(2-4x)^2}$$

Plug the formula for y into the result of part (a) to see if the same answer is obtained.

$$y' = \frac{2\left(\frac{x}{2-4x}\right)^2}{x^2} = \frac{2}{x^2}\frac{x^2}{(2-4x)^2} = \frac{2}{(2-4x)^2}$$